

farmer regard the investigator as his necessary helper in the conduct of his business.

In matters connected with the physics of the soil and its bearing upon the operations of cultivation the American workers have accumulated much novel information, and to this some of the chapters of Prof. Snyder's book form a good introduction. The requirements of the crop are treated from a sound general standpoint which never forgets that water and air, soil texture, and cultivation are perhaps the prime factors in plant production. In this country students are a little too apt to fancy that farming begins and ends with the application of artificial manures; we can recommend this book to them for the truer point of view, even though the conditions which regulate our use of manures are not quite the same as in America.

#### RECENT ASPECTS OF ELEMENTARY GEOMETRY.

*The First Book of Geometry.* By Grace Chisholm Young, Ph.D., and W. H. Young, M.A., Sc.D. Pp. xvi+222. (London: J. M. Dent and Co., 1905.) Price 1s. 6d. net.

OF late years a very remarkable change has been made in the theory of elementary geometry, the general effect of which has been to make it more abstract, and to reduce a great deal of it to the application of logic without any appeal to intuition. It has been realised that geometry must be based on the assumption of certain undefinable entities, of elementary relations between them, and a complete system of independent axioms. For the purposes of ordinary Euclidean geometry, it is probably the simplest way to assume the straight line as the one undefinable entity, and intersection as the elementary relation from which the notions of point and plane may be derived. What system of axioms we adopt will partly depend upon the nature of the geometry we study; for instance, the axioms which are necessary and sufficient for the purposes of projective geometry require supplementing when we discuss the theory of measurement.

It is the theory of measurement which presents the greatest difficulty at the present time. If we assume all the results of projective geometry, we may proceed as follows:—Taking any three points O, I, X on a line, we may associate them with the numbers (or indices) 0, 1,  $\infty$  (where  $\infty$  is the vague infinity of ordinary arithmetical algebra). We can then give a purely projective rule for finding a point on the line to be associated with any given rational number  $p/q$ ; we thus get on the line a set of points corresponding to the whole field of rational numbers, and, moreover, the arrangement of the points corresponds to the arrangement of the numbers according to their magnitude; that is, if  $m > n > p$ , the point N lies on that segment MP which does not contain X. If we like, we can define the distance AB as being measured by  $b-a$ , where  $a, b$  are the indices of A, B. This satisfies the relation  $AB+BC=AC$ , but equal segments as thus defined are not intuitively equal,

except when X is "the" point at infinity on the line; and even then we cannot prove, but must assume the intuitional equality. Moreover, there are points on the line which do not have rational indices, unless, in spite of common sense, we assume that the points on the line form a discrete aggregate. Now in arithmetic we have a perfect continuous aggregate, where each irrational element separates all the rational ones into two complementary parts, respectively greater or less than itself. If we assume that all the points in the line which have not rational indices behave in a similar way, we have a complete correspondence between the succession of points on a line and the elements of the arithmetical continuum. So far as appears at present, this is a pure assumption; but if it is not made, anything like the ordinary theory of measurement seems to be impossible, for two distinct points ought to have a measurable distance, and the measure must be a number; if the two distinct points cannot be associated with two distinct numbers, how is their distance to be defined as a measurable quantity? Other difficulties arise in connection with transfinite numbers and their representation by point aggregates; but these are comparatively unimportant, if it is remembered that the assumption of the correspondence of points on a line with the arithmetical continuum involves a similar correspondence between the arithmetical continuum and the points on any finite segment.

It is very interesting to see how this recent theory has reacted on the question of teaching elementary geometry. Instead of tending to make it more abstract and more logical, it has done exactly the reverse; and the reason for this is not difficult to find. The notions of geometry, so far as it is distinct from logic, are derived from concepts, and these, again, from experience. There must be an intuitional basis for geometry; and although, from a logical point of view, it is desirable, for any given species of geometry, to reduce its necessary assumptions to a minimum, progress in geometrical invention is to be expected from those who cultivate their powers of observation as well as their logical faculties. One result of recent research has been to explode, once for all, the pretence that the "Elements" of Euclid present geometry in its most logical form; on the other hand, to try to teach beginners the subject in what would now be considered the most rigorous way would be certain to end in failure.

The book which has been written by Dr. and Mrs. Young illustrates very well what has just been said. Its main object is to awaken the pupil's mind to the ideas by which we classify the properties of space; this is done by directions in paper-folding, in dissection of areas, in the construction of solid models, and the like. At the same time, various theorems are stated and proved, so that the beginner may learn the difference between experimental and deductive geometry. As in the case of other text-books with a similar aim, the teacher will have to be careful to see that his pupil distinguishes proofs from verifications; e.g., on p. 173 we have a proof that the angles of a triangle make up two right angles, while on p. 121 we have a verification in a special case.

There is no doubt that the kind of first course laid out in this book is the right one from a psychological point of view. A quite young pupil, actually carrying out its directions with the help of a sympathetic teacher, will obtain clear conceptions of geometrical facts in a way that is really interesting and fruitful. The apparatus required is of the simplest possible kind—paper, pins, a pencil, and a pair of scissors are all that are absolutely necessary, though a compass and a scale might be provided with advantage, except at the beginning of the course. The attention given to solid figures is a feature which deserves commendation; and above all there does not seem to be anything said that is likely to lead to misapprehensions, which have to be painfully corrected afterwards.

There are a few points of detail which might be attended to in another edition. The letters in the diagrams are too small; the figure on p. 151 does not correspond to the text; on p. 35, "This gives us another axiom" is quite illogical; and there are some technical terms which might have been spared. It must be remembered, too, that this is not a book for a beginner to *learn* in the old-fashioned way; it is intended to make him experiment and think, and the guidance of the teacher is essential. Assuming this, the book ought to be very useful, and lead to good results, even in the case of pupils who have little faculty for demonstrative geometry.

G. B. M.

#### LEGIBILITY AND VISUAL ACUITY.

*Physiologie de la Lecture et de l'Écriture.* By Emile Javal. Pp. xv+296. (Paris: Félix Alcan, 1905.) Price 6 francs.

THE title of this book, which is written by a distinguished ophthalmologist, is somewhat misleading. One would naturally expect such a work to deal with the neuro-muscular mechanism, central and peripheral, of reading and writing. In reality, it treats almost exclusively of the legibility of printed and written matter, and the physiological processes are investigated only in so far as they throw light upon this aspect of the subject, and give indications for increased facility and rapidity in reading.

In the first part of the book a brief historical account of epigraphy, writing, typography, stenography, musical notation, and writing in relief for the blind is given. Typography is illustrated by examples from Garamond (1540), adopted by Plantin, of Antwerp, and the two Elzeviers, of Leyden and Amsterdam respectively, from very elegant designs by Jaugeon (1704), and from the Imprimerie Impériale (Didot, 1811) and the Imprimerie Nationale (Marcellin Legrand, 1847). Theoretical considerations of visual acuity, treated in the second part, show that the visibility of a letter increases indefinitely with the illumination, whereas its legibility depends upon the neuro-epithelial mosaic of the retina, and is therefore independent of illumination above a certain minimum. Investigation of the mechanism of reading a line of print has shown that the eyes move in a series of

jerks, in each of which a group of about ten letters is appreciated, the grouping being independent of the distance of the book from the eyes so long as this is consistent with legibility. Bar reading gives some indication of the relative part played by the two eyes when binocular vision is present. The difference of accommodation in different parts of a line when the book is held close to the eyes, as in myopia, is very appreciable, and must be taken into account in treatment; thus in a myope of 15 dioptries a line of 10 centimetres involves a difference of accommodation of about 7 dioptries. The characteristic features of letters are for the most part in the upper portion, so that attention is specially directed here; consequently it is easy to read with the lower halves of the letters covered, whilst the reverse occasions considerable difficulty. These considerations indicate some improvements in typography. They have been carried out in some designs prepared for the author by M. Ch. Dreyfuss. It will be admitted that the result is successful as regards legibility and rapidity of reading, though at no small cost to the artistic sense. M. Javal points out that nearly all the improvements are to be seen in the well known enamelled-iron advertisement of Willing; indeed, English printing as a whole compares favourably in his estimation with that of other countries.

The terrible misfortune of blindness overtook the author a few years ago, so that it is not surprising that he has given much attention to Braille type. Even those born blind rarely attain to a rate of 100 words a minute in reading, or 10 in writing. It is surprising to find that the tactile acuity of the blind is actually less than that of normal people; the reading finger tires rapidly, and though the acuity of other fingers is greater, they are comparatively useless for reading. The author gives valuable suggestions for improving and simplifying Braille type, as well as general instructions as to the hygiene of vision and of writing.

J. HERBERT PARSONS.

#### OUR BOOK SHELF.

*Exercises in Quantitative Chemistry.* By Harmon Northrop Morse. Pp. xx+356. (Boston and London: Ginn and Co., 1905.) Price 8s. 6d.

THE time when the sole desideratum in the training of the chemical student was the acquirement of greater or less proficiency in the processes of analytical chemistry has, happily, gone by. Courses of experimental work arranged with the view of familiarising him with the most important general reactions, the preparation of typical organic compounds, and the methods peculiar to physical chemistry are now recognised as the essentials of chemical training. The work under review has been written from this standpoint, and makes no attempt to present a course of work for the training of expert analysts.

Although much diversity of opinion must necessarily attach to the problem of the choice of an ideal course of exercises, the unbiased critic can have but little fault to find with the author's selection. At first sight the heterogeneous character of the sixty-four exercises creates an impression of a lack of systematic arrangement, but this is more apparent than real. The first eight chapters deal with the balance, the